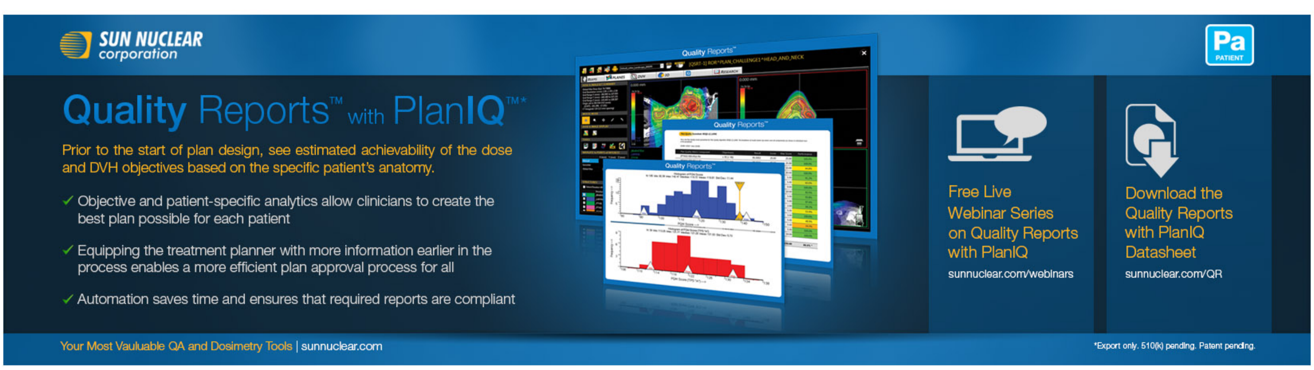
Medical Physics
The International Journal of Medical Physics Research and Practice

# A method for comparing beamhardening filter materials for diagnostic radiology

Robert J. Jennings

Citation: Medical Physics **15**, 588 (1988); doi: 10.1118/1.596210
View online: http://dx.doi.org/10.1118/1.596210
View Table of Contents: http://scitation.aip.org/content/aapm/journal/medphys/15/4?ver=pdfcov
Published by the American Association of Physicists in Medicine





# A method for comparing beam-hardening filter materials for diagnostic radiology

Robert J. Jennings
*Office of Science and Technology, Center for Devices and Radiological Health, Rockville, Maryland 20857*

(Received 20 August 1987; accepted for publication 19 April 1988)

The necessity for using adequate beam filtration in diagnostic radiology is well known. Although aluminum is the most widely used filter material for diagnostic x-ray applications, the possibility that other materials might have superior properties has prompted a number of studies that have attempted to determine both the type and the amount of filtration most appropriate for a given situation. This paper describes a method based on precise matching of spectral shape that permits the absolute ranking of beam-hardening materials. Matching of spectral shape ensures equality of such parameters as image contrast and patient dose. Spectrally equivalent filters can then be ranked on the basis of the transmission of one relative to another. Following the development of the theory behind the method and an algorithm for implementing it, the method is applied to the evaluation of a variety of materials for use as filters in diagnostic radiology. Experimental verification of a few of the calculated results is also described. Both calculated and experimental results show that normal aluminum filters are about 10% less efficient than filters of materials such as copper, brass, or iron. Since the approach followed here was the basis for several early investigations of filtration for orthovoltage therapy, a brief comparison of results from these early reports with results calculated using the method developed here is also presented.

Key words: x-ray filters, Thoraeus filters, x-ray spectra, filter efficiency

## I. INTRODUCTION

It has been recognized for over 50 yr that filter materials can be compared on the basis of two different criteria, quantitative equivalence and qualitative equivalence.[1,2] Two filters are quantitatively equivalent if they produce the same decrement in exposure, as measured by an ionization chamber type device, for a specified incident x-ray spectrum. Quantitatively equivalent attenuators do not, in general, transmit equal relative intensity distributions (spectra). From this it can be shown that quantitative equivalence is dependent on the incident spectrum, a fact that is well known and that limits the usefulness of the concept.

According to the definition of Thoraeus,[1] two filters are qualitatively equivalent if they transmit the same relative spectral intensity distribution for a given incident spectrum. This is a strict definition for it requires that the transmissions of the two filters be in the same ratio at all energies for which the transmitted fluence rates are non-negligible. The value of the qualitative equivalence approach to the comparison of filters is that such filters can be ranked quantitatively on the basis of a single figure of merit, the ratio of the transmission of one to the transmission of the other, which by the definition is independent of energy. All other filter-dependent parameters, such as entrance skin exposure required to produce an image, contrast, and relative depth dose will be the same. Moreover, once qualitative equivalence has been demonstrated for a reference spectrum, it must hold for any other spectrum contained within the bounds of the reference spectrum.

The relevance of the qualitative equivalence approach depends on the existence of filter pairs that satisfy the definition. Although it is not immediately obvious that one can find pairs of filters made from different materials that are qualitatively equivalent by the strict definition, it is a simple matter to find pairs of filters that satisfy the more relaxed definition of producing the same half-value layer (HVL) for a given incident spectrum. Such filter pairs are good candidates for qualitative equivalence by the strict definition since strictly equivalent pairs, when used to filter the same spectrum, must produce transmitted spectra having the same HVL. The less stringent definition has been widely used, and was the basis for selecting filters to be compared on the basis of other properties in the work of Mayneord and Roberts,[2] and Trout *et al.*[3] All of the investigators that have been mentioned attempted to verify the qualitative equivalence of the filters they selected for study, Thoraeus by calculating transmission curves from tabulated attenuation coefficients,[1] Mayneord and Roberts by comparing photographic spectra recorded using a Seeman spectrograph,[2] and Trout *et al.* by calculating spectra from transmission curves.[3]

The development of high-resolution, energy dispersive x-ray detectors (high-purity germanium, for example) has greatly improved the accuracy with which x-ray spectra can be measured, thus permitting precise comparison of the spectra transmitted by different filters. In an experiment to compare copper and aluminum by this method, it was observed that the shape of the spectrum transmitted by an aluminum filter could be reproduced identically, to within experimental uncertainty, by a copper filter of appropriate thickness. It was also observed that the intensity of the beam transmitted by the copper filter was greater than the intensity of the beam transmitted by the aluminum filter, and that the advantage of copper over aluminum increased with filter thickness. That experiment and a brief analysis of it have been reported.[4] The spectra recorded for the experiment have been reported by Siedband.[5] The theoretical development that follows is a generalized and more complete version of the analysis in Ref. 4.

## II. THEORY

Let the narrow-beam transmission of two filters be given by $T_1$ and $T_2$, with corresponding attenuation coefficients $\mu_1$ and $\mu_2$, and thicknesses $t_1$ and $t_2$. The situation observed experimentally can be described mathematically by

$$T_2/T_1 = \exp(\mu_1 t_1 - \mu_2 t_2) = K. \quad (1)$$

The attenuation coefficients $\mu_1$ and $\mu_2$ depend on energy but $K$ does not, at least over the range of the spectrum observed in the experiment. Taking the natural logarithm and energy derivative of both sides of Eq. (1) yields

$$\mu_2' t_2 - \mu_1' t_1 = \left(\frac{-1}{K}\right)\frac{dK}{dE} = 0$$

or

$$\frac{t_2}{t_1} = \frac{\mu_1'}{\mu_2'}, \quad (2)$$

where

$$\mu' \equiv \frac{d\mu}{dE}.$$

An alternative way of looking at Eq. (2) is that, even if the existence of a match in spectral shape over a large energy range has not been verified, adjusting $t_2$ for a given value of $t_1$ so that Eq. (2) is satisfied at some energy $E$ forces $dK/dE$ to be zero at that energy. This guarantees a spectral shape match at that energy and in the neighborhood of that energy. This match is a result not of the energy dependences of the processes by which x rays interact with matter but rather of the property of the exponential function $\exp(-\mu t)$ that, when differentiated, the thickness $t$ appears as a parameter that scales the derivative. Since no condition has been imposed on absolute transmissions, the ratio $t_2/t_1$ can be treated as an adjustable parameter that can be selected to provide a spectral match.

By using Eq. (2) to substitute for either $t_1$ or $t_2$ in Eq. (1), it can be seen that $K$ depends exponentially on filter thickness. For example, if a pair of filters has a relative efficiency $K$ of 1.1, doubling the thickness of each filter will result in a relative efficiency of 1.21. Depending on the materials involved, $K$ can assume large values for thick filters. Thus, it is precisely in the circumstance that one wishes to increase filtration substantially that the choice of material is most important. For any combination of materials, $K$ tends to unity as filter thickness approaches zero.

A simple example shows that the efficiency of any filter is directly related to the rate of change of its transmission with energy. In a case where one filter exhibits transmission values of 0.1 and 0.9 at two given energies, and a second filter has transmission values of 0.1 and 0.3 at the same energies, doubling the thickness of the second filter will produce transmissions in the same ratio as the transmissions of the first filter but smaller by an order of magnitude. Clearly, the material whose attenuation coefficient decreases most rapidly with energy is the most efficient.

To relate the preceding example to the relative performance of different materials, it is necessary to examine the energy and atomic number dependences of the physical phenomena involved in the interaction of x rays with matter. The photoelectric coefficient varies rapidly with energy, approximately as $1/E^3$, while the Compton coefficient is nearly independent of energy. The magnitude of the photoelectric coefficient also varies rapidly with atomic number, approximately as $Z^3$, while the magnitude of the Compton coefficient depends only weakly on $Z$. (The coherent scattering coefficient is not considered in this argument because it is substantially smaller that the photoelectric coefficient at energies where the photoelectric effect is dominant and only becomes comparable to it at energies where the Compton effect dominates.) Thus the efficiency of a material as a filter will increase with atomic number up to the point where the photoelectric effect dominates the behavior of the material at all energies in the range of interest. Beyond that value of $Z$, all materials for which the $K$-edge energy remains below the energy range of interest will be very nearly equivalent. The limitation to using higher atomic number materials is that eventually the $K$-edge discontinuity in the photoelectric effect coefficient enters the useful range of energies, and the filter transmits photons at energies below its $K$ edge. To use a material that has its $K$ edge in the energy range of interest, it is necessary that it be thick enough to have negligible transmission below its $K$ edge, or that it be used with an auxiliary filter of lower atomic number that will suppress the low-energy transmission of the main filter. The lower $Z$ material will also absorb the fluorescence radiation from the higher $Z$ filter.

The selection of filter thickness in accordance with Eq. (2) guarantees a good spectral match in the neighborhood of the energy at which the ratio $\mu_1'/\mu_2'$ is evaluated. Qualitative equivalence, however, requires a good match over an entire spectrum, so it is necessary to look at the accuracy of the spectral match between two materials over a large energy range. One approach is to examine the behavior of $\mu_1'/\mu_2'$ as a function of energy. If it is constant, then the ratio of thicknesses required for a spectral match is constant, and a perfect match in spectral shape between filters of the two materials can be achieved. Figure 1 shows a plot (solid curve) of $\mu_1'/\mu_2'$ vs energy for the range 10 to 150 keV, with aluminum as material 1 and copper as material 2. The maximum and minimum values of $\mu_1'/\mu_2'$ differ by more than a factor of 2. For purposes of comparison, $\mu_1/\mu_2$, which is the ratio of thicknesses required for equal transmission at a given energy, is also plotted in Fig. 1 (dashed curve).

Since the spectral match between copper and aluminum is very good in the energy range examined,[5] even though the value of $\mu_1'/\mu_2'$ is clearly not constant, it is of interest to examine analytically the error in the spectral match as a function of energy when the thickness ratio $t_2/t_1$ has been fixed. The appropriate quantity to examine is not the ratio $T_2/T_1$ but the difference in transmissions $(T_2/K_0) - T_1$, where $K_0$ is the value of $K$ at the energy at which an exact match is desired. The reason is related to the fundamental nature of a filter, which is to preferentially remove only a certain part of the spectrum. Only differences that are significant relative to the passband transmissions of the filters are

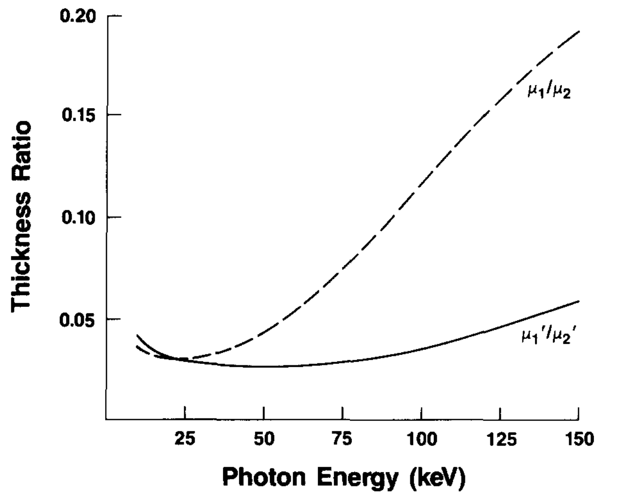



FIG. 1. Plot of the ratio of thicknesses of aluminum and copper filters providing spectral shape matching (solid curve) and equal transmission (dashed curve), as functions of energy.

important. For example, if two filters have transmissions that differ by an order of magnitude at an energy at which both have very low transmissions, say $10^{-3}$ and $10^{-4}$, while their transmissions at some other energy are matched and are of the order of unity, the difference between them in a practical sense is negligible, but the error in their transmission ratio is large. On the other hand, two filters that have transmissions matched at an energy at which the transmissions are small, and differ by several percent at an energy at which the transmission is high are sensibly different but will have a much smaller error in transmission ratio than the previous pair.

The approach taken here is to examine the energy derivative of the transmission difference. This expression, by the fundamental theorem of calculus, is the integrand of the integral over energy that gives the transmission difference at any energy. Study of the behavior of this integrand will make clear the reason that the transmission error is less than might be expected from the behavior of $\mu_1'/\mu_2'$.

We define $R = \mu_1'/\mu_2'$, $R_0 = R(E_0) = t_2/t_1$, and $K_0 = K(E_0)$. The transmission error $\mathscr{E}_T$ is given by

$$\mathscr{E}_T = (T_2/K_0) - T_1. \tag{3}$$

By taking the energy derivative of both sides of Eq. (3) and using the definitions above, it can be shown that

$$\frac{d\mathscr{E}_T}{dE} = \mu_1' t_1 T_2 \left(\frac{K_0}{K} - \frac{R_0}{R}\right)\Big/ K_0. \tag{4}$$

We note first that the derivative is zero at $E_0$ since the term in parentheses vanishes. This should not be a surprise since the derivative evaluated at $E_0$ is the coefficient of the first-order term in the Taylor's series expansion of the transmission difference. The definition of $K_0$ and the procedure used to select $t_2$ set this coefficient to zero. Also, variations in the thickness ratio, represented by $R_0/R$ (the normalized inverse of the function plotted as the solid curve in Fig. 1), are in part compensated by changes in $K_0/K$ since changes in both are of the same sign. For example, if $R$ increases, indicating that $t_2$ would have to be increased to provide a perfect spectral match, $K$ must also increase, since $t_2$ being too small corresponds to $T_2$ being too large. Second, as energy is decreased from the match point, $T_2$ becomes very small. Finally, as energy is increased from the match point, $\mu_1$ becomes dominated by the Compton effect, which is only weakly dependent on energy, and its derivative $\mu_1'$ becomes very small. Thus the integrand of the error integral is a product of three energy-dependent terms, each of which becomes small in a different energy region, with the result that the agreement between the two filters at both ends of the energy range of interest may be expected to be considerably better than the "thickness fraction" curve (solid curve, Fig. 1) would indicate. The quantitative verification of these qualitative observations will be discussed following the description of a numerical filter matching algorithm based on the preceding theory.

## III. NUMERICAL ALGORITHM

A numerical algorithm has been developed to determine the best match between two filters and to evaluate the goodness of fit between their transmission curves. The first step in the algorithm is the specification of the composition and thickness of filter 1 and the composition of filter 2. Attenuation coefficients and their derivatives are generated for materials 1 and 2 using the ln $\mu$ vs ln $E$ polynomial fits provided in the compilation of tabulated data of McMaster *et al.*[6] The sum rule is used if either material has more than one element. Then $t_2/t_1$ is calculated at user-specified intervals in a user-specified range of energies (see Fig. 1).

The next step in the algorithm is to determine the value of $t_2/t_1$ that results in the minimum root-mean-square (rms) transmission error in the matching of the two filters over the specified energy range. Following the treatment above, the transmission error is taken to be the difference between the actual transmission of one filter and the scaled transmission of the other. One estimate of the scaling factor could be obtained by evaluating Eq. (1) at the energy at which $t_2/t_1$ was determined. However, the value obtained in this way would probably not result in the best match, in an rms sense, between the two filters. Instead, the error for a given value of $E$ (and therefore of $t_2/t_1$) is calculated in the following way:

(1) calculate transmissions $T_1$ and $T_2$ at every energy in the range specified:

$$T_i(E) = \exp[-\mu_i(E)t_i], \quad i = 1,2; \tag{5}$$

(2) calculate the average value of $T_2/T_1$, weighted by $T_1$, i.e.,

$$K_{av} = \frac{\Sigma_E[(T_2/T_1)T_1]}{\Sigma_E T_1} = \left(\frac{T_2}{T_1}\right)_{av}; \tag{6}$$

(3) calculate the rms error

$$\mathscr{E}_{rms} = \left(\frac{\Sigma_E(1 - T_2/T_1K_{av})^2 T_1^2}{\Sigma_E T_1^2}\right)^{1/2}; \tag{7}$$

for every one of the $N$ values of $t_2/t_1$ that has been considered. The minimum value, and corresponding values of $E$, $t_2/t_1$, and $K_{av}$ are saved.

Specification of the energy range, which determines the limits over which the sums in steps 2 and 3 are performed, serves as a coarse means of adjusting the algorithm to produce a good match for a particular spectrum. A better match can be obtained by replacing the weighting term $T_1$ in step 2 by $T_1\ S$, where $S$ represents the spectrum. In this case, $\mathscr{E}_{\text{rms}}$ is given by

$$\mathscr{E}_{\text{rms}} = \left(\frac{\Sigma_E[(1 - T_2/T_1K_{\text{av}})T_1S]^2}{\Sigma_E(T_1S)^2}\right)^{1/2}. \quad (8)$$

The result is, of course, less general.

As an example, the algorithm has been used to find the best match between copper and a reference filter of 3.0 mm of Al. A 100-kVcp x-ray beam with 2.5 mm of Al filtration, from the compilation of Fewell *et al.*[7] was used as the weighting spectrum. The values of $t_2/t_1$, $t_2$, and $K_{\text{av}}$ determined by the algorithm are 0.027 16, 0.081 47 mm, and 1.1172, respectively. The value of $t_2/t_1$ selected by the algorithm corresponds to 36 keV. The rms error in the match is 0.36%. To illustrate the energy dependence of the matching of the two filters, $T_1$, $T_2$, and $T_2/K_{\text{av}}$ have been plotted in Fig. 2. A plot of $T_2/T_1$ is shown in Fig. 3, and filtered spectra $T_1S$ and $(T_2/K_{\text{av}})S$ have been plotted in Fig. 4. Although the deviation of $T_2/T_1$ from the average value $K_{\text{av}}$ becomes large at low energies (Fig. 3), the small size of both $T_1$ and $T_2$ results in very small differences in $\mathscr{E}_T$, and thus in transmission (Fig. 2) or spectra (Fig. 4) in the same energy range.

To verify the qualitative observations made earlier about the behavior of $d\mathscr{E}_T/dE$ and $\mathscr{E}_T$, the energy-dependent factors in Eq. (4) have also been calculated for the matching of the 3.0-mm aluminum filter by copper. In this example, uniform weighting from 10 to 150 keV was used. Figure 5 is a plot of $-\mu_1'(E)$ as a function of energy. (The minus sign renders the quantity positive since $\mu$ is a decreasing function of energy.) The transmission $T_2$ is shown in Fig. 2. The product $-\mu_1' T_2$ is plotted in Fig. 6. The influence of $T_2$ forces the product to zero at low energy, and the influence of $\mu_1'$ causes it to approach zero at higher energies. The quantity $(K_0/K - R_0/R)$ is shown in Fig. 7. As predicted, it is zero at the match energy $E_0$, which again is 36 keV. Both $d\mathscr{E}_T/dE$ and $\mathscr{E}_T(E)$ are shown as solid lines in Fig. 8. The oscillations in $d\mathscr{E}_T/dE$ prevent the accumulation of large values for $\mathscr{E}_T(E)$. These quantities have been recalculated using $K_{\text{av}}$ in place of $K_0$, and are plotted as the dashed curves in Fig. 8. Shifting from $K_0$ (1.1161) to $K_{\text{av}}$ (1.1106) reduces the rms error from 0.70% to 0.47%. The value of $\mathscr{E}_T$ is less than 0.01 everywhere in the 10- to 150-keV energy range considered.

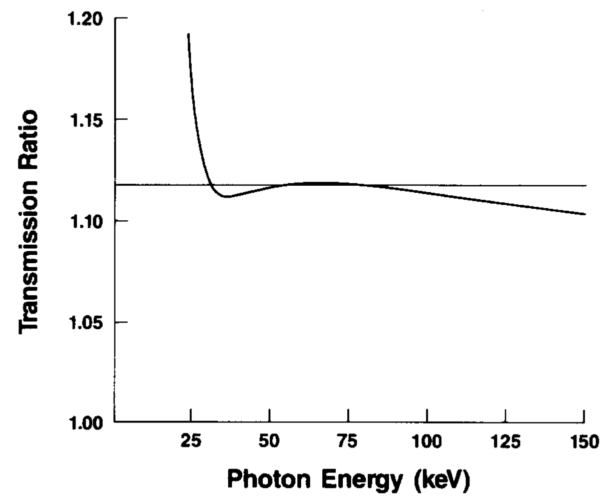



FIG. 3. Plot of the ratio of the transmission of a 0.081 47-mm Cu filter to the transmission of a 3.0-mm Al filter. The horizontal line is drawn at a value of 1.1172, the value of the scaling factor $K_{\text{av}}$ that yields the minimum rms difference in 100-kVp spectra transmitted by the two filters.

## IV. CALCULATED RESULTS

The algorithm just described has been used to compare a number of materials. For purposes of discussion, 3.0 mm of Al have been selected as the reference filter. The same mate-

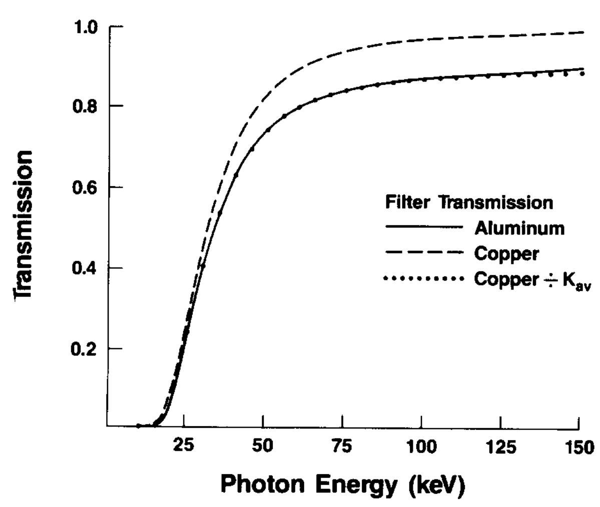



FIG. 2. Transmission as a function of energy for 3.0 mm Al (solid curve), and 0.081 47 mm Cu (dashed curve). The third graph (dotted curve) is the transmission curve for Cu divided by 1.1172.

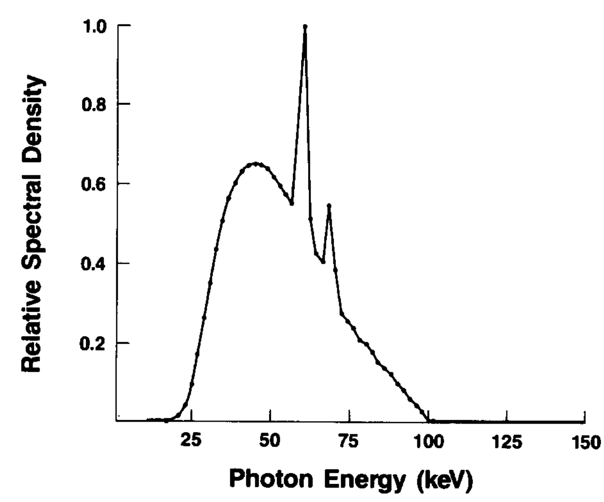



FIG. 4. Plots of 100-kVp spectra transmitted by 3.0 mm of Al (solid curve) and 0.081 47 mm of Cu (dotted curve). The Cu filtered spectrum has been divided by the scaling factor 1.1172.

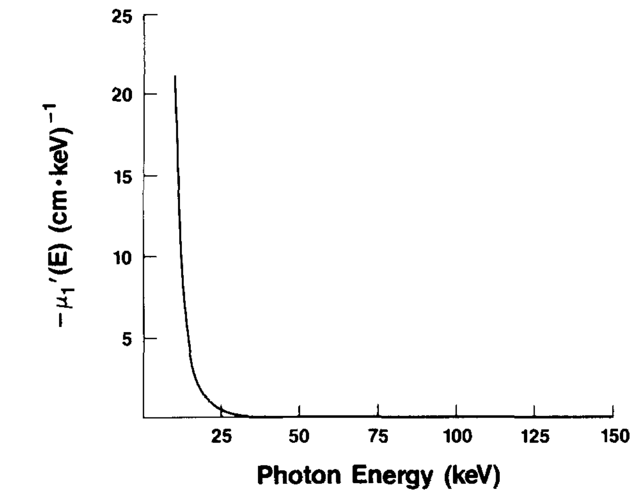



FIG. 5. Plot of $-\mu'(E)$ as a function of energy for aluminum.

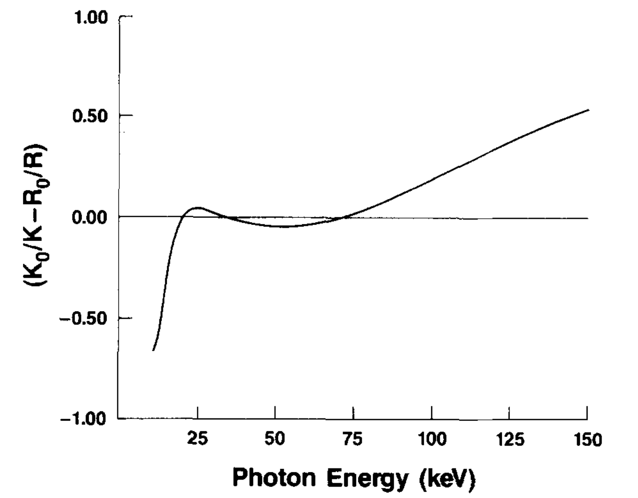



FIG. 7. Plot of the quantity $(K_0/K - R_0/R)$ as a function of energy [see text, Eq. (4), for explanation].

rials have also been compared using copper as the reference filter. The thickness, 0.083 37 mm, produced the best spectral match to the 3.0-mm Al filter using uniform weighting over the range 10 to 150 keV. Table I shows thickness ratio $R_0$, best-fit transmission ratio $K_{av}$, and rms error $\mathscr{E}_{rms}$ for each material relative to aluminum for 10- to 150-keV uniform weighting ($K$ edge to 150-keV uniform weighting for materials with $K$ edges above 10 keV) and for weighting by 65-, 100-, and 140-kVp spectra.[7,8] Table II gives the same parameters determined using copper as the reference filter.

Several conclusions can be drawn from these tables. First, for quartz, chosen as a reasonable approximation of glass, and all materials below it in the tables (higher in atomic number), the spectral match to aluminum is very good, as shown by the low values of $\mathscr{E}_{rms}$. Moreover, $t_2/t_1$ and $K_{av}$ are nearly independent of the weighting spectrum used, which is in agreement with the low values of $\mathscr{E}_{rms}$ for these materials for the uniform 10- to 150-keV weighting, which are all less than 1%. Consequently, for these materials, a single thickness ratio can be chosen that will provide a good spectral match for any spectrum produced at a tube potential of 150 kV or less. In fact, for some pairs of materials, the spectral match holds up to much higher energies. The general qualitative equivalence of beam-hardening filter materials is summarized in Fig. 9, which is a plot of $\mathscr{E}_{rms}$ as a function of atomic number for matching to the copper reference filter by elements with atomic numbers from 3 to 50. Uniform weighting from 10 to 150 keV, or from the $K$ edge to 150 keV for atomic numbers from 31 to 50, was used. The large values of $\mathscr{E}_{rms}$ for very low atomic number materials indicate that they are simply not spectrally equivalent to aluminum in thicknesses sufficient to provide useful filtering properties.

However, Eq. (4) indicates an approximately linear dependence of $\mathscr{E}_T$ on filter thickness, leading to the expectation that a good match in spectral shape is achievable for

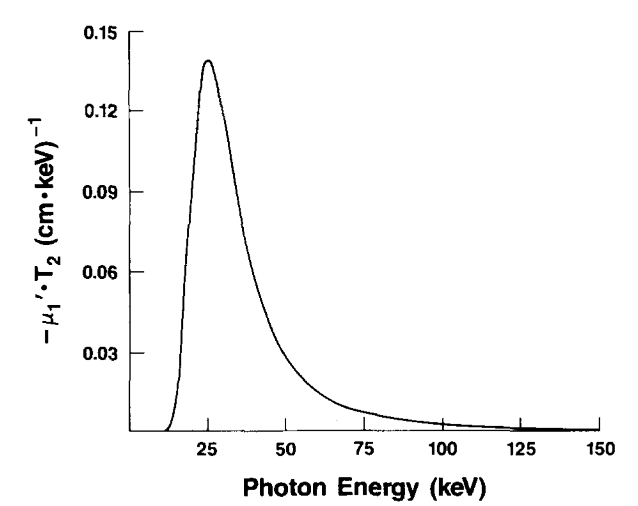



FIG. 6. Plot of $-\mu_1'(E)T_2$ as a function of energy. Subscripts 1 and 2 refer to aluminum and copper, respectively.

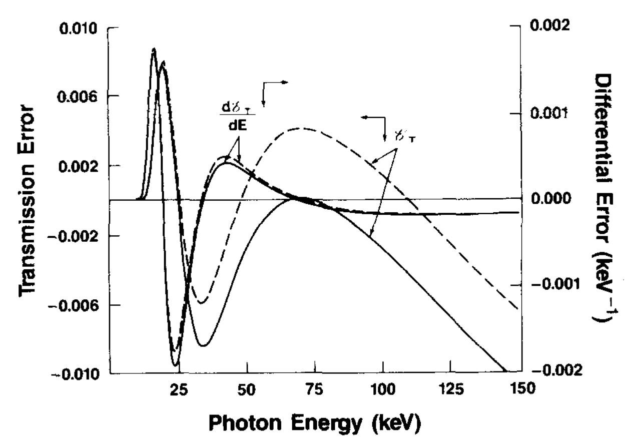



FIG. 8. Plots of transmission error $\mathscr{E}_T$ [Eq. (3) in text] and its energy derivative $d\mathscr{E}_T/dE$ [Eq. (4)] as functions of energy, plotted as solid curves. These quantities have been recalculated with $K_{av}$ in place of $K_0$ and are plotted as dashed curves.

TABLE I. Performance of filter materials relative to aluminum: $U^*$ indicates uniform weighting from 10 to 150 keV, except as noted. Calculations based on an aluminum thickness of 3.0 mm.

| Spectrum (kVp) | | 65 | | | 100 | | | 140 | | | $U^*$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Atomic No. | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ |
| 4 Be | 57.13 | 0.0096 | 5.36% | 38.63 | 0.0526 | 7.17% | 29.98 | 0.1122 | 7.88% | 23.41 | 0.2104 | 12.1% |
| 6 C | 20.85 | 0.2057 | 2.10% | 17.66 | 0.2883 | 3.46% | 15.84 | 0.3445 | 4.18% | 12.87 | 0.4653 | 6.53% |
| PMMA ($C_5H_8O_2$, $\rho = 1.19$) | 19.43 | 0.3068 | 1.65% | 16.85 | 0.3899 | 2.82% | 15.40 | 0.4409 | 3.46% | 12.90 | 0.5486 | 5.46% |
| Quartz ($SiO_2$, $\rho = 2.65$) | 1.427 | 0.9415 | 0.04% | 1.420 | 0.9453 | 0.15% | 1.416 | 0.9473 | 0.22% | 1.392 | 0.9558 | 0.42% |
| 12 Mg | 2.016 | 0.9606 | 0.10% | 2.022 | 0.9592 | 0.06% | 2.022 | 0.9596 | 0.07% | 2.005 | 0.9633 | 0.17% |
| 14 Si | 0.8957 | 1.0325 | 0.11% | 0.901 1 | 1.0287 | 0.13% | 0.903 0 | 1.0275 | 0.14% | 0.9080 | 1.0242 | 0.22% |
| 23 Va | 0.08231 | 1.1046 | 0.31% | 0.081 71 | 1.1079 | 0.18% | 0.081 52 | 1.1074 | 0.17% | 0.08334 | 1.1004 | 0.41% |
| 26 Fe | 0.04189 | 1.1115 | 0.41% | 0.041 42 | 1.1166 | 0.23% | 0.041 31 | 1.1160 | 0.20% | 0.04241 | 1.1081 | 0.47% |
| 28 Ni | 0.02923 | 1.1125 | 0.48% | 0.028 84 | 1.1188 | 0.28% | 0.028 78 | 1.1182 | 0.22% | 0.02951 | 1.1109 | 0.48% |
| 29 Cu | 0.02767 | 1.1083 | 0.58% | 0.027 16 | 1.1172 | 0.36% | 0.027 05 | 1.1173 | 0.25% | 0.02779 | 1.1106 | 0.47% |
| 30 Zn | 0.03071 | 1.1116 | 0.57% | 0.030 24 | 1.1192 | 0.34% | 0.030 14 | 1.1191 | 0.24% | 0.03097 | 1.1121 | 0.41% |
| 39 Y (18–150 keV) | 0.02605 | 1.0924 | 1.21% | 0.024 95 | 1.1147 | 0.85% | 0.024 63 | 1.1177 | 0.57% | 0.02552 | 1.1130 | 0.79% |
| 47 Ag (26–150 keV) | 0.00720 | 1.0760 | 1.18% | 0.006 81 | 1.1052 | 1.02% | 0.006 72 | 1.1099 | 0.77% | 0.00673 | 1.1120 | 0.63% |
| 50 Sn (30–150 keV) | 0.01137 | 1.0831 | 0.71% | 0.010 81 | 1.1071 | 0.73% | 0.010 70 | 1.1106 | 0.58% | 0.00833 | 1.1133 | 0.42% |

even very low-$Z$ materials relative to aluminum or copper if the thickness is sufficiently small. Trivially, the match is exact ($\mathscr{E}_T$ is zero and $K_0$ unity) when filter thickness is zero. The good match found by Nagel between 30 mm of Be and 0.365 mm of aluminum is a nontrivial example.[9] Thus the spectral equivalence approach provides an accurate means of specifying in terms of a standard material the beam-shaping properties and relative throughput of low-$Z$ materials of thicknesses typically encountered in x-ray tube construction (Be, or insulating oil and plastic, for example).

To examine the goodness of fit for various spectra using a single filter thickness ratio, we compared 3.0 mm of Al to 0.083 37 mm of Cu. Values of $K_{av}$ and $\mathscr{E}_{rms}$ for five different weighting spectra, data for which are given in Refs. 7 and 8, are shown in Table III. The matching errors increase only slightly compared to those in Tables I and II. The maximum variation in $K_{av}$ amounts to 0.3% from a mean value of 1.1094.

Second, as anticipated in the qualitative discussion of filter behavior, lower atomic number materials have a "throughput" deficit relative to higher $Z$ materials. The deficit decreases as $Z$ increases, and in the case of the 0.083 37 mm Cu reference filter becomes less than 1% for vanadium, which has an atomic number of 23. This information is summarized in Fig. 10, where $K_{av}$ relative to the Cu reference filter has been plotted as a function of atomic number for atomic numbers from 3 to 50. Again, uniform weighting from 10 keV or the $K$-edge energy to 150 keV was used. Since the efficiency of one filter relative to another depends exponentially on thickness, the deficit shown by the lower $Z$ mate-

TABLE II. Performance of filter materials relative to copper: $U^*$ indicates uniform weighting from 10 to 150 keV, except as noted. Calculations based on a copper thickness of 0.083 37 mm.

| Spectrum (kVp) | | 65 | | | 100 | | | 140 | | | $U^*$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Atomic No. | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ | $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ |
| 4 Be | 2104 | 0.0077 | 4.85% | 1455 | 0.0405 | 7.04% | 1078 | 0.1012 | 8.02% | 840.2 | 0.1904 | 12.2 % |
| 6 C | 758.4 | 0.1816 | 1.61% | 647.7 | 0.2516 | 3.29% | 581.9 | 0.3013 | 4.26% | 470.2 | 0.4127 | 6.72% |
| PMMA ($C_4H_8O_2$, $\rho = 1.19$) | 700.3 | 0.2767 | 1.17% | 621.1 | 0.3401 | 2.64% | 568.4 | 0.3853 | 3.52% | 473.2 | 0.4860 | 5.66% |
| Quartz ($SiO_2$, $\rho = 2.65$) | 51.56 | 0.8491 | 0.56% | 52.21 | 0.8432 | 0.30% | 52.21 | 0.8448 | 0.34% | 50.05 | 0.8608 | 0.79% |
| 12 Mg | 72.75 | 0.8672 | 0.68% | 74.52 | 0.8552 | 0.41% | 74.83 | 0.8550 | 0.30% | 72.18 | 0.8672 | 0.63% |
| 13 Al | 36.14 | 0.9010 | 0.58% | 36.82 | 0.8928 | 0.36% | 36.97 | 0.8923 | 0.25% | 35.98 | 0.9004 | 0.47% |
| 14 Si | 32.44 | 0.9303 | 0.68% | 33.15 | 0.9193 | 0.47% | 33.40 | 0.9175 | 0.32% | 32.67 | 0.9222 | 0.46% |
| 23 Va | 2.979 | 0.9961 | 0.27% | 3.007 | 0.9916 | 0.19% | 3.017 | 0.9907 | 0.14% | 3.002 | 0.9907 | 0.19% |
| 26 Fe | 1.513 | 1.0032 | 0.17% | 1.524 | 0.9996 | 0.15% | 1.528 | 0.9987 | 0.12% | 1.528 | 0.9976 | 0.15% |
| 28 Ni | 1.056 | 1.0040 | 0.10% | 1.061 | 1.0017 | 0.10% | 1.062 | 1.0011 | 0.09% | 1.063 | 1.0001 | 0.11% |
| 30 Zn | 1.111 | 1.0027 | 0.01% | 1.112 | 1.0022 | 0.02% | 1.112 | 1.0020 | 0.03% | 1.113 | 1.0015 | 0.04% |
| 39 Y (18–150 keV) | 0.9419 | 0.9854 | 0.64% | 0.9183 | 0.9977 | 0.51% | 0.9109 | 1.0002 | 0.30% | 0.9183 | 1.0023 | 0.53% |
| 47 Ag (26–150 keV) | 0.2622 | 0.9689 | 0.83% | 0.2511 | 0.9895 | 0.80% | 0.2492 | 0.9929 | 0.67% | 0.2442 | 1.0009 | 0.66% |
| 50 Sn (30–150 keV) | 0.3301 | 0.9705 | 0.54% | 0.3157 | 0.9899 | 0.63% | 0.3110 | 0.9947 | 0.57% | 0.3028 | 1.0020 | 0.52% |

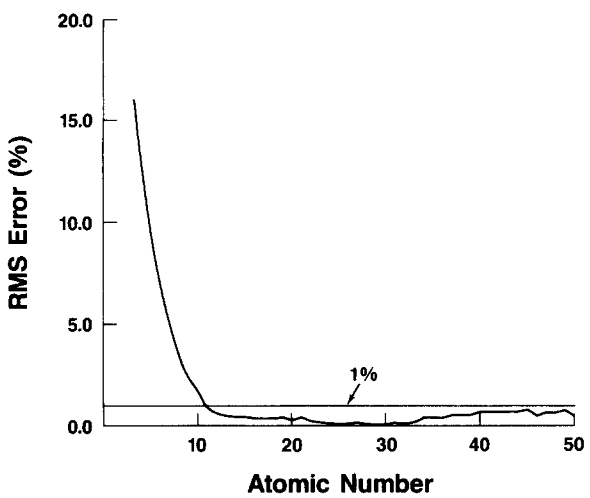



FIG. 9. Plot of spectral match error $\mathscr{E}_{rms}$ for elements with atomic numbers from 3 to 50, relative to a 0.083 37-mm Cu filter, as a function of atomic number.

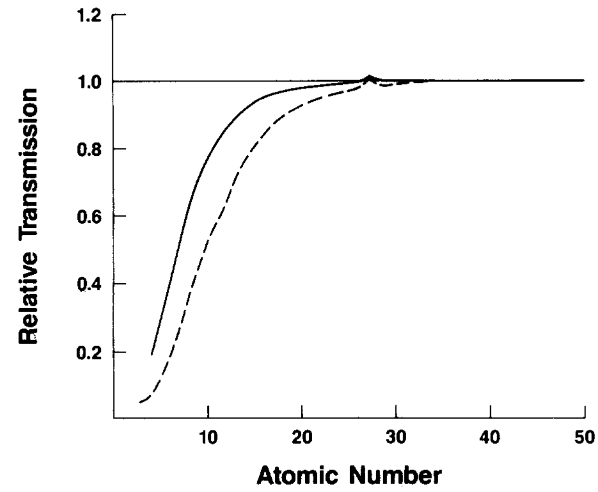



FIG. 10. Plot of the ratio of transmissions $K_{av}$ for elements from 3 to 50 relative to a 0.083 37-mm Cu reference filter as a function of atomic number (solid curve). Also shown is $K_{av}$ for the same elements relative to a 0.25-mm Cu filter (dashed curve). The second curve has been renormalized by dividing by 1.014 (see text).

rials will be greater for thicker reference filters, and the value of $Z$ for which the deficit becomes negligible will be greater. This is shown by a second plot in Fig. 10, of $K_{av}$ relative to a 0.25-mm Cu reference filter. These data have been renormalized by dividing by 1.014, the average of the $K_{av}$ values relative to the copper reference filter for atomic numbers from 36 to 50. The first element to be within 1% of the asymptotic value of $K_{av}$ is zinc, which has an atomic number of 30.

Finally, comparison of relative efficiencies and thickness ratios from Tables I and II demonstrates that the errors in the spectral matching of copper and aluminum are sufficiently small that reasonably accurate values of $K_{av}$ and $R_0$, for a third material relative to copper for instance, can be calculated from Table I using the data for copper and the third material relative to aluminum. Conversely, values relative to aluminum can be calculated from Table II using the data for aluminum and the third material relative to copper. To give an example, $R_0$ and $K_{av}$ for Mg and Cu relative to Al for the case of uniform weighting are 2.005 and 0.9633, and 0.027 79 and 1.1106, yielding values for Mg relative to Cu of 72.15 for $R_0$ and 0.8674 for $K_{av}$. The corresponding values from Table II are 72.18 and 0.8672. By extension, reasonably accurate values for any pair of materials for which $\mathscr{E}_{rms}$ is small can be obtained from either table.

## V. EXPERIMENTAL VERIFICATION

High-resolution, energy-dispersive measurements of x-ray spectra have been used to test the numerical method for finding and evaluating qualitatively equivalent filters. The measurements were made using high-purity germanium detectors and multichannel analyzers (MCA's). In one set of experiments, which have been previously reported,[4,5] two thicknesses of copper were compared to experimentally determined best match thicknesses of aluminum. An 86-kVp spectrum was used for the test. The first filter tested, 0.10 mm of Cu, was matched by 3.7 mm of Al. The second filter, 0.25 mm of Cu, was matched by 9.3 mm of Al. The smallest increment in aluminum thickness available was 0.1 mm. In both cases the transmitted spectra matched very closely.[5]

When the spectra were measured, data accumulation was terminated when a predetermined number of counts had been recorded. The relative filter efficiency could then be calculated from the MCA live times required to acquire a pair of spectra. Table IV shows the experimentally determined values of $K_{av}$ for the two sets of filters along with values calculated for the same filter thicknesses using a 90-kVp weighting spectrum. Although the agreement between experimental and calculated values is not perfect, especially for the thicker filter pair, it is within the experimental uncertainty resulting from x-ray generator output variations and imprecision in the measurement of the thicknesses of the filters that were used.

The filter matching algorithm predicts slightly different thicknesses for the best match aluminum filters than were used experimentally. When adjustment was made in order to calculate $K_{av}$ for the experimental thicknesses, it was found that although $K_{av}$ changed, the error in the spectral match was only slightly affected. The implication of this is that it should be reasonably easy to achieve a spectral match between two filter materials experimentally, but the observed relative efficiency of the filters may differ from the values in the tables.

In another experiment, which was performed to evaluate claims for the superiority of the filtration properties of yttrium (Y) compared to other materials,[10] spectra transmit-

TABLE III. Comparison of spectral matching parameters determined using uniform weighting with best-fit parameters, for several spectra.

| kVp | Uniform weighting $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ | Best fit $t_2/t_1$ | $K_{av}$ | $\mathscr{E}_{rms}$ |
|---|---|---|---|---|---|---|
| 65 | 0.027 79 | 1.1059 | 0.59% | 0.027 67 | 1.1083 | 0.58% |
| 80 | 0.027 79 | 1.1079 | 0.60% | 0.027 41 | 1.1136 | 0.49% |
| 100 | 0.027 79 | 1.1105 | 0.56% | 0.027 16 | 1.1172 | 0.36% |
| 120 | 0.027 79 | 1.1112 | 0.51% | 0.027 14 | 1.1171 | 0.28% |
| 140 | 0.027 79 | 1.1114 | 0.46% | 0.027 05 | 1.1173 | 0.25% |

TABLE IV. Experimental measurement of efficiencies of two copper filters relative to spectrally equivalent aluminum filters.

| Filter | Counting times (s) | Time ratio | $K_{av}$ (theory) | Error |
|---|---|---|---|---|
| 0.1 mm Cu<br>3.7 mm Al | 390<br>451 | 1.156 | 1.148 | 0.7% |
| 0.25 mm Cu<br>9.3 mm Al | 180<br>248 | 1.378 | 1.416 | 2.8% |

ted by filters of 0.10 mm Y, 0.11 mm Cu, and 4.08 mm Al were measured and compared. The incident spectrum was nominally 80 kVp, with a 1.5-mm Al filter added to the inherent tube filtration. Measurements were made both with and without a phantom that simulates the attenuation of the human chest.[11]

Spectra without the phantom are shown in Fig. 11, and spectra with the phantom are shown in Fig. 12. All plots have been normalized to have the same area. Additional data for the six spectra are given in Table V. The values listed include the total number of counts recorded, tube current, counting time (MCA live time), percent dead time of the pulse pile-up rejection system that was used, calculated HVL's of the spectra, and the actual kVp, observed with voltage dividers, during data collection. The table also lists both experimental and calculated filter efficiencies, relative to the aluminum filter, for the copper and yttrium filters. The experimental values have been corrected for differences in the dead time of the pulse pile-up rejection system and in tube current. As the HVL values and the spectra themselves demonstrate, the spectral match is good. Agreement between experimental and calculated relative filter efficiencies is also good. These data thus confirm the validity of the theory, as well as the numerical algorithm based on it, for materials with a wide range of atomic numbers (13, 29, and 39 for Al, Cu, and Y, respectively). These data, along with results from Tables I and II, also clearly show that yttrium has no special advantage over materials such as copper or iron, contrary to the conclusions of Wang *et al.*[10] A more detailed analysis of their report has been given elsewhere.[12]

## VI. COMPARISON WITH PREVIOUS RESULTS

A considerable body of work exists on the efficiency of qualitatively equivalent filters for orthovoltage therapy, exemplified by the work of Thoraeus,[1,13] Mayneord and Roberts,[2] and Trout, *et al.*[3] In this section we compare calculations performed using the approach described above with previously published results.

Thoraeus, in his initial published report on the subject of qualitatively equivalent filters,[1] gave three conditions for the qualitative equivalence of x-ray spectra: (1) same minimum wavelength (maximum energy); (2) same maximum wavelength (minimum energy); and (3) same relative spectral intensity distribution. Condition (1), as he stated, is simply the requirement that the spectra be generated at the same tube potential. Assuming no differences in high-voltage waveforms, condition (3) is given in mathematical terms by Eq. (1). In considering condition (2), Thoraeus recognized that, rigorously, one cannot speak of an end point for x-ray spectra since x-ray attenuation is exponential. He therefore proposed the practical definition of minimum energy as that energy for which the filter transmission is $10^{-3}$. He used this definition to calculate thicknesses of filters to be used in experiments. Condition (2) can be written as

$$\exp(-\mu_1 t_1) = \exp(-\mu_2 t_2) = 10^{-3}.$$

This leads directly to the condition that

$$t_2/t_1 = \mu_1/\mu_2,$$

which is inconsistent with Eq. (1). However, the require-

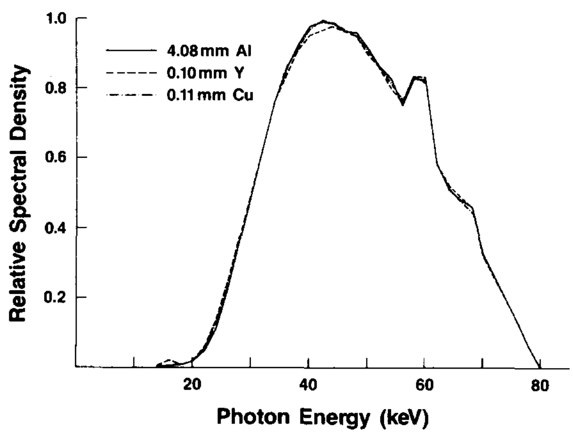



FIG. 11. Spectra transmitted by filters of 0.10 mm Y, 0.11 mm Cu, and 4.08 mm Al. The incident spectrum was produced at 80 kVp, and filtered by 1.5 mm of Al in addition to the inherent filtration of the tube. All curves have been normalized to have the same area.

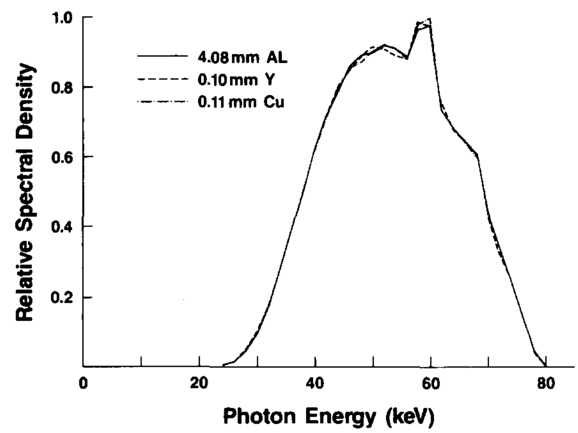



FIG. 12. The same spectra shown in Fig. 11 but attenuated by a phantom simulating the attenuation of an average chest. The curves have been normalized to the same area as in Fig. 11.

TABLE V. Spectroscopic comparison of aluminum, copper, and yttrium filters.

| Filter | kVp | mAs | HVL mm Al | Dead time | Total counts | Corr. counts | $K_{av}$ (exp.) | $K_{av}$ (theory) |
|---|---|---|---|---|---|---|---|---|
| No Phantom | | | | | | | | |
| 4.08 mm Al | 80.3 | 1.22 | 4.56 | 4% | $3.955\times10^5$ | $4.120\times10^5$ | ... | ... |
| 0.11 mm Cu | 80.3 | 1.22 | 4.50 | 5% | $4.514\times10^5$ | $4.751\times10^5$ | 1.153 | 1.167 |
| 0.10 mm Y | 80.4 | 1.22 | 4.43 | 5% | $4.508\times10^5$ | $4.745\times10^5$ | 1.152 | 1.172 |
| With Chest Phantom | | | | | | | | |
| 4.08 mm Al | 80.3 | 1.72 | 6.46 | 4% | $2.039\times10^5$ | $2.124\times10^5$ | ... | ... |
| 0.11 mm Cu | 80.3 | 1.72 | 6.45 | 5% | $2.327\times10^5$ | $2.450\times10^5$ | 1.154 | 1.164 |
| 0.10 mm Y | 80.3 | 1.73 | 6.44 | 5% | $2.328\times10^5$ | $2.436\times10^5$ | 1.147 | 1.161 |

ment that the transmission be $10^{-3}$ implies large values of $\mu$, assuming reasonable filter thicknesses, or in other words, that $\mu_1/\mu_2$ be evaluated at low energies. Figure 1, which shows both $\mu_1/\mu_2$ and $\mu_1'/\mu_2'$ plotted as functions of energy demonstrates that these functions are closest for low energies. Thus even though Thoraeus' maximum wavelength condition was not valid, it resulted in the selection of filter thicknesses that were close to correct.

In the same paper Thoraeus presented experimental results for the comparison of copper with aluminum, and tin with copper.[1] In the case of copper and aluminum, he calculated that 0.5 mm of Cu and 16.0 mm of Al would be spectrally equivalent. In comparing calculated transmission curves for these filters, he found them to be in the ratio of 1.74 to 1. Our algorithm predicts a thickness of 16.7 mm of Al to match 0.5 mm of Cu, with the ratio of transmissions, $K_{av}$ equal to 1.75. The value of $K_{av}$ giving the best agreement between 16.0 mm of Al and 0.5 mm of Cu is 1.68. Thoraeus' experiments showed the ratio of transmissions of these filters to be 1.40.

Although Thoraeus reported both calculated and experimental data for a number of tin-based filters,[1,12] results will be compared only for the one he designated as the "normal" tin filter, which consisted of 0.44 mm Sn + 0.25 mm Cu + 1.0 mm Al. Thoraeus' original calculation[1] indicated that the "normal" filter would be qualitatively equivalent to 2.0 mm Cu + 1.0 mm Al. Using a 165-kVp spectrum, he found that it was matched by 2.1 mm Cu + 1.0 mm Al, with a relative efficiency of 1.30. He later reported that the "normal" filter was equivalent to 2.06 mm Cu + 1.0 mm Al, with a relative efficiency of 1.25. On removing the common elements of the two filter sets, Thoraeus' finding results in the claim that 0.44 mm of Sn is spectrally equivalent to 1.81 mm of Cu. We find, using uniform weighting from 40 to 150 keV instead of a 165-kVp spectrum, that 0.44 mm of Sn is qualitatively equivalent to 1.62 mm of Cu, with a relative efficiency of 1.14. On the other hand, $K_{av}$ for 0.44 mm of Sn relative to 1.81 mm of Cu is 1.225, and $\mathscr{E}_{rms}$ for this combination is 3.2%. We conclude that, given the error in Thoraeus' method of calculating equivalent filter thicknesses and the relative insensitivity of the spectral match to exact thickness, there is reasonable agreement between our computational method and his results for thicknesses of filters that are qualitatively equivalent. In comparing values of relative efficiency $K_{av}$ found by Thoraeus and by our method, agreement is again good, except for the experimental value found by Thoraeus in comparing Cu and Al. In that case, his own calculated value also disagreed with the experimental result.

Shortly after the publication of the papers of Thoraeus,[1,13] Mayneord and Roberts published a study entitled "The 'Quality' of High Voltage Radiations."[2] They discussed quantitative and qualitative equivalence of filters in much the same way as Thoraeus, and concluded that, from a practical point of view, equality of HVL was the best alternative available for selecting qualitatively equivalent filters. Although they investigated several materials, copper and tin are the only ones for which relative efficiencies for qualitatively equivalent pairs were given.

The data were presented as curves of percent improvement for tin relative to copper as a function of HVL at three different tube voltages. Corresponding filter thicknesses are available from their plots of HVL versus filter thickness. We have selected one point from each of the three efficiency curves for comparison with calculations done using the algorithm described above. Their data and our results are listed for each tube potential in Table VI.

Since spectra at the high voltages considered here were not available for use in weighting our calculations, we used uniform weighting over an interval somewhat smaller than the range of the spectra used by Mayneord and Roberts. The range, and the step size within that range, are given in the table. Since our "best match" filter thicknesses are different

TABLE VI. Thoraeus filters: Comparison of theory with data of Mayneord and Roberts.

| Spectrum | Copper filter | Thoraeus filter | $K_{av}$ | $\mathscr{E}_{rms}$ |
|---|---|---|---|---|
| 200 kVp | 2.0 mm | 0.5 mm Cu + 0.43 mm Sn | 1.19 ... | |
| 50–180 keV in | 1.5 mm | 0.407 mm Sn[a] | 1.13 | 0.3% |
| 1-keV steps | 1.5 mm | 0.43 mm Sn[b] | 1.10 | 1.4% |
| 300 kVp | 3.0 mm | 0.5 mm Cu + 0.625 mm Sn | 1.38 | |
| 50–280 keV in | 2.5 mm | 0.69 mm Sn[a] | 1.20 | 0.7% |
| 2-keV steps | 2.5 mm | 0.625 mm Sn[b] | 1.24 | 2.2% |
| 380 kVp | 4.0 mm | 0.5 mm Cu + 0.70 mm Sn | 1.41 | |
| 50–340 keV in | 3.5 mm | 0.997 mm Sn[a] | 1.28 | 1.2% |
| 2-keV steps | 3.5 mm | 0.70 mm Sn[b] | 1.40 | 7.0% |

[a] Values for best spectral match.
[b] Values obtained using Sn thickness used by Mayneord and Roberts.

than the ones found by Mayneord and Roberts to give equal HVL's, we also include in the table results obtained by forcing the algorithm to use their thicknesses. Values of $\mathscr{E}_{\rm rms}$ are included to give an indication of the degree to which the spectral match is compromised by doing this.

Examination of Table VI shows that the agreement in tin thickness for a given thickness of copper is good at tube potentials of 200 and 300 kV, but the efficiencies predicted by our method do not agree very well with the values found by Mayneord and Roberts. At 380 kV, just the opposite is true. The tin thicknesses are substantially different, but the relative efficiency calculated by our method for the tin thickness used by Mayneord and Roberts matches very closely the value they found experimentally. Despite these quantitative differences, the qualitative behavior they observed is in agreement with that predicted by our analysis in that the relative advantage of tin over copper increases with filter thickness.

In 1961, Trout *et al.* applied the results of their recently completed investigation into improved techniques for determining HVL[14] to a reexamination of the relation between Thoraeus filters and copper filters.[3] As was the case with previous workers, they reported efficiencies of Thoraeus filters relative to copper filters as a function of HVL. They reported data for three tube potentials, 200, 250, and 300 kVp. Several examples that were discussed in the paper have been selected for comparison. Their data and our calculations are compared in Table VII, in a manner analogous to Table VI. Except for the case of the heaviest filter at 300 kVp, the agreement in filter thickness for pairs of filters yielding equal spectra is very good, and demonstrates the importance of eliminating the effects of scatter in measurements of HVL. There are consistent differences between our calculated ratios of the transmission of Thoraeus filters relative to copper and the measured data of Trout *et al.* Our calculated values are all higher than the experimental data. Most of the differences are in the range of 20% to 35% when calculated in terms of the advantage of the Thoraeus filter over copper, or 5% to 15% when taken as errors in transmission.

TABLE VII. Thoraeus filters: Comparison of theory with data of Trout *et al.*

| Spectrum | Copper filter | Thoraeus filter | $K_{\rm av}$ | $\mathscr{E}_{\rm rms}$ |
|---|---|---|---|---|
| 200 kVp | 2.0 mm | 0.25 mm Cu + 0.51 mm Sn | 1.05 | |
| 50–180 keV in 1-keV steps | 1.75 mm | 0.47 mm Sn[a] | 1.15 | 0.3% |
| | 1.75 mm | 0.51 mm Sn[b] | 1.11 | 2.0% |
| 250 kVp | 4.1 mm | 0.25 mm Cu + 1.10 mm Sn | 1.25 | |
| 50–230 keV in 2-keV steps | 3.85 mm | 1.06 mm Sn[a] | 1.34 | 0.6% |
| | 3.85 mm | 1.10 mm Sn[b] | 1.32 | 1.2% |
| 300 kVp | 2.7 mm | 0.25 mm Cu + 0.68 mm Sn | 1.13 | |
| 50–280 keV in 2-keV steps | 2.45 mm | 0.68 mm Sn[c] | 1.20 | 0.7% |
| 300 kVp | 4.0 mm | 0.25 mm Cu + 1.06 mm Sn | 1.23 | |
| 50–280 keV in 2-keV steps | 3.75 mm | 1.06 mm Sn[c] | 1.31 | 0.9% |
| 300 kVp | 7.8 mm | 0.25 mm Cu + 2.0 mm Sn | 1.55 | |
| 50–280 keV in 2-keV steps | 7.55 mm | 2.19 mm Sn[a] | 1.70 | 1.4% |
| | 7.55 mm | 2.00 mm Sn[b] | 1.79 | 2.8% |

[a] Values for best spectral match.
[b] Values obtained using Sn thickness used by Trout *et al.*
[c] Best spectral match thickness equals experimental thickness.

## VII. DISCUSSION

The results of our calculations indicate that materials such as copper or iron are preferable to other beam-hardening filters in the normal diagnostic x-ray energy range. Since the advantage of these materials over aluminum in the normal range of thicknesses is only about 10%, it seems unlikely that replacement of filters in existing equipment is warranted. However, in applications where heavier filtration is desired, the use of copper or iron is definitely worthwhile. When these materials are selected for use, 0.5 mm of Al filtration should be retained, downstream from the main filter, to suppress fluorescence radiation and transmission at energies below the $K$ edge of the main filter. For applications involving beryllium window tubes, iron is preferred over copper. The $K$ edge of copper, at 8.98 keV, is just above the $L_\alpha$ lines of tungsten at 8.33 and 8.40 keV, so that the aluminum equivalence of a copper filter for these lines is a factor of 10 less than at slightly higher energies.

The recent work of Koedooder and Venema[15] indicates that iron, copper, etc., may also be preferable to $K$-edge filters. They considered the case of imaging iodine in water, and chose to hold contrast and tube loading constant. Filters were compared on the basis of dose reduction. In those cases where $K$-edge filters did perform better, the advantage was only a few percent. If, as seems likely, this result also holds for the imaging of other targets, then the question of the best filter material for general diagnostic radiology may be put to rest and effort can be put into other, more productive areas of research. Practical considerations will continue, however, to make nonoptimal materials useful when they have other desirable but unrelated properties. Transparent, lead-loaded acrylic plastic is an example. Add-on filters of this material can be used for the occasional exam requiring extra filtration done in a busy room since they can be added outside the collimator without interfering with the collimator light field. In this application they produce a useful improvement when the alternative, if one were limited to "optimal" materials, would be no improvement at all.

A few calculations have also been performed for the mammographic energy range. Good spectral matching can also be achieved in this energy range, but filter thickness ratios depend more strongly on the spectrum or energy range in question and on the amount of filtration being considered, so no data have been tabulated. However, it seems unlikely that aluminum will be displaced as the filter material of choice for mammography by another beam-hardening filter material. At 45 kVp, replacing 3.0 mm of Al filtration (taken as an upper limit for filtration in xeromammography) by 0.5 mm of Al and the iron equivalent of the remaining 2.5 mm of Al results in a reduction in tube loading of only 7%. At lower tube voltages and with less Al filtration, the advantage of other materials is negligible.

A situation similar to that observed by Koedooder and Venema for the normal diagnostic energy range appears to obtain for mammography as well. Muntz *et al.*,[16] in calculations to determine the minimum dose configuration for a mammographic imaging system with fixed imaging performance, considered aluminum as well as a variety of *K*-edge filters. Aluminum filtration always resulted in the lowest dose configuration. Calculated data recently reported by Stanton *et al.*[17] seem to be consistent with this observation.

The spectral matching approach presented here has applications beyond the evaluation of beam-hardening filters. It can, for example, be used in evaluating the attenuation properties of materials commonly used as tissue substitutes. To illustrate, consider the use of polymethylmethacrylate (PMMA, also known as Lucite, Plexiglas, Perspex) as a substitute for breast tissue. Using data from Hammerstein *et al.*[18] for the composition of 50% adipose/50% glandular breast (H, 0.107; C, 0.4015; N, 0.0245; O, 0.464; P, 0.001; S, 0.002; density, 0.985), and the formula and density ($C_5H_8O_2$, 1.19) for PMMA, and using uniform weighting over the energy range 10 to 50 keV, our algorithm indicates that 4.50 cm of breast tissue is spectrally equivalent to 4.477 cm of PMMA. The transmission of the PMMA relative to the breast tissue is 0.863. The spectral match in this case is very good, $\mathscr{E}_{\rm rms}$ being only 0.2%. Information of this type may be useful not only in selecting materials for tissue simulation but also in comparing results of experiments done with phantoms of different compositions.

Another application involves *K*-edge filters. Good matches can be achieved between beam-hardening materials and *K*-edge filters for energy ranges that have the *K* edge as either their upper or lower limit. Although we arrived at this conclusion independently, it has been discussed previously by Chan *et al.*[19] and Bäuml.[20] We have discussed an application of this aspect of the spectral matching technique to the generation of subtracted spectra that may be useful for dual-energy imaging.[21]

One aspect of the use of beam filters that our approach does not address directly is the influence of scatter generated in the filter. The importance of scatter from the filter was clearly demonstrated by the work of Trout *et al.*[14] Ardran and Crooks[22] studied filter-generated scatter under conditions relevant to diagnostic equipment by performing experiments in which added aluminum filtration was placed at different positions within the tube collimator. Recently, Karellas *et al.*[23] have argued that the relative amount of scatter generated should be the criterion used in comparing filter materials. While we feel that the criterion of maximum throughput is more relevant, it should be pointed out that this criterion implies the selection of the material with the most rapidly varying attenuation coefficient, and therefore the largest ratio of photoelectric to scattering coefficient, consistent with *K*-edge and fluorescence radiation constraints. Thus it is optimal with regard to the criterion of Karellas *et al.* as well.

## VIII. CONCLUSIONS

We have presented a method based on precise matching of spectral shape that permits the absolute ranking of beam-hardening materials on the basis of a straightforward, objective criterion, their relative transmission. We have demonstrated that the required spectral matching can be achieved under circumstances that are sufficiently general that the approach can be applied to most situations in diagnostic radiology and orthovoltage therapy. We have presented experimental verification of our calculated results, and have shown that our calculational method produces results that are in reasonable agreement with earlier work on the evaluation of Thoraeus filters. We have also touched on some of the other possible applications of the idea of spectral matching. Although this concept goes back to the work of Thoraeus,[1,13] it has received little attention from researchers in diagnostic radiology, with the exception of the recent work of Nagel.[9] The concept is very useful, however, because it can be applied independent of the details of the spectra under consideration, a property that the concept of attenuation equivalence does not enjoy, and because of the insight it provides.

## ACKNOWLEDGMENTS

The author gratefully acknowledges the careful experimental work of T. R. Fewell, who performed the spectroscopic comparison of Cu, Fe, and Y filters. The original derivation of the equal spectrum condition [Eq. (2)] was done by R. J. Eastgate. The author gratefully acknowledges useful discussions with H. D. Nagel, who provided a prepublication manuscript (Ref. 9) on his closely related work, and is responsible for the observation concerning the difficulty in using Cu filters with Be window tubes. (The mention of commercial products herein is not to be construed as either an actual or implied endorsement of such products by the Department of Health and Human Services.)

[1]R. Thoraeus, "A study of the ionization method for measuring the intensity and absorption of Roentgen rays and of the efficiency of different filters used in therapy," Acta Radiol., Suppl. **15** (1932).

[2]W. V. Mayneord and J. E. Roberts, "The 'quality' of high voltage radiations. I.," Br. J. Radiol. **8**, 341 (1935).

[3]E. D. Trout, J. P. Kelley, and A. C. Lucas, "Evaluation of Thoraeus filters," Am. J. Roentgenol., Radium Ther. Nucl. Med. **85**, 933 (1961).

[4]R. J. Jennings, "X-ray filters and beam quality," in *Quality Assurance in Diagnostic Radiology*, AAPM Medical Physics Monograph No. 4, edited by R. G. Waggener and C. R. Wilson (American Institute of Physics, New York, 1980), pp. 110–112.

[5]M. P. Siedband, "Beam filtration in diagnostic radiology," in *Real-Time Radiologic Imaging: Medical and Industrial Applications*, edited by D. A. Garrett and D. A. Bracher, ASTM Special Technical Publication 716 (ASTM, Philadelphia, PA, 1980), pp. 159–160.

[6]W. H. McMaster, N. K. Del Grande, J. H. Mallett, and J. H. Hubbell, "Compilation of x-ray cross sections," University of California, Lawrence Radiation Laboratory Report No. UCRL-50174, Sec. II, Rev. 1 (NTIS-1969).

[7]T. R. Fewell, R. E. Shuping, and K. R. Hawkins, Jr., *Handbook of Computed Tomography X-Ray Spectra*, HHS Publication (FDA) 81-8162, April 1981. (Spectra at 100, 120, and 140 kVp listed as EI4, EI9, and EI8, respectively, in the reference.)

[8]T. R. Fewell and R. E. Shuping, "Photon energy distributions of some typical diagnostic x-ray beams," Med. Phys. **4**, 187 (1977) (65- and 80-kVp spectra).

[9]H. D. Nagel, "Aluminum equivalence of materials used in diagnostic radiology and its dependence on beam quality," Phys. Med. Biol. **31**, 1381 (1986).

[10]Y. Wang, G. H. McArdle, S. A. Feig, S. Karasick, H. A. Koolpe, E. Mapp, V. M. Rao, R. M. Steiner, and R. J. Wechsler, "Clinical application of yttrium filters for exposure reduction," RadioGraphics **4**, 479 (1984).

[11]B. J. Conway, P. F. Butler, J. E. Duff, T. R. Fewell, R. E. Gross, R. J. Jennings, G. H. Koustenis, J. L. McCrohan, F. G. Rueter, and C. K. Showalter, "Beam quality independent attenuation phantom for estimating patient exposure from x-ray automatic exposure controlled chest examinations," Med. Phys. **11**, 827 (1984).

[12]J. A. Nelson and R. J. Jennings, "Critical review: 'Clinical application of yttrium filters for exposure reduction'," Invest. Radiol. **21**, 601 (1986).

[13]R. Thoraeus, "Tin-filters in roentgen therapy," Acta Radiol. **15**, 225 (1934).

[14]E. D. Trout, J. P. Kelley, and A. C. Lucas, "Determination of half-value layer," Am. J. Roentgenol., Radium. Ther. Nucl. Med. **84**, 729 (1960).

[15]K. Koedooder and H. Venema, "Filter materials for dose reduction in screen–film radiography," Phys. Med. Biol. **31**, 585 (1986).

[16]E. P. Muntz, H. Jafroudi, R. J. Jennings, and H. Bernstein, "An approach to specifying a minimum dose system for mammography using multiparameter optimization techniques," Med. Phys. **12**, 5 (1985).

[17]L. Stanton, J. L. Day, T. Villafana, C. H. Miller, and D. A. Lightfoot, "Screen–film mammography technique for breast cancer screening," Radiology **163**, 471 (1987).

[18]G. R. Hammerstein, D. W. Miller, D. R. White, M. E. Masterson, H. Q. Woodard, and J. S. Laughlin, "Absorbed radiation dose in mammography," Radiology **130**, 485 (1979).

[19]J. L.-H. Chan, R. E. Alvarez, and A. Macovski, "Measurement of soft tissue overlying bone utilizing broad band energy spectrum techniques," IEEE Trans. Nucl. Sci. **23**, 551 (1976).

[20]A. Bäuml, "Überlegungen zum Problem der äquivalenten Filterung in der Röntgendiagnostik" and "Vorschläge zu ihrer Bestimmung aus dem Röntgenspektrum," Report STH 23/77 Neuherburg: Institut für Strahlenhygiene das Bundesgesundheitsamtes (1979).

[21]R. J. Jennings, "Synthetic x-ray spectra for dual energy imaging," Med. Phys. **12**, 536 (1985).

[22]G. M. Ardran and H. E. Crooks, "Dose in diagnostic radiology: The effect of changes in kilovoltage and filtration," Br. J. Radiol. **35**, 172 (1962).

[23]A. Karellas, N. Tam, M. A. Davis, and C. J. D'Orsi, "Absorption and scattering properties of elemental filters for x-ray beams," Proc. SPIE **626**, 170 (1986).